Mispricing and trading strategies in Chicago Board Options Exchange

Siyi Fan[1], Chen Ai[2]

[1]Faculty of Natural & Mathematical Sciences, King's College London, London WC2R 2LS, UK

[2]College of Science, Nanchang University, Nanchang, Jiangxi 330002, China

Keywords: trading strategies, price violations, Chicago Board Options Exchange, Black-Scholes model

Abstract: In this paper, the importance of detecting the mispricing in option market, and some methods to test the mispricing which includes the boundary violation, convexity and exercise price violation, the relationship between option lives and prices violation, put-call parity and Black-Scholes model (BSM) together with implied volatility in Chicago Board Options Exchange were also studied. Trading strategies to exploit the mispricing were also suggested in this paper. Based on our data, more violations were found on call options except for strike price violation. Also, the violation frequency of the options in aspect of boundary violation was found to be sensitive to the stock price. Only little attention was recommended to be puton the violation of convexity and time to maturity due to less profit compared with transaction cost. However, the violations of exercise price were considered to be relatively serious given the reason of certain high degree of mispricing based on the theoretical price. With reference to put-call parity and BSM violations, they were both frequently detected and should be seriously considered. And for BSM, most of the pricing errors of call options appeared with larger moneyness, while most of the pricing errors of put options appeared with smaller moneyness. In conclusion, more attention should be suggested to be put on exercise price conditions, put-call parity and BSM violations.

1. Introduction

The option, in finance, is the contract which empowers the buyer the right to buy or sell the underlying asset at a specific strike price at or prior to the maturity based on the type (American or European). Since it is highly correlated to the volatility of the stock market, and unlike the futures, it depends on the choice of buyer that whether to exercise the option, it can be used to generate extremely large profit or doom the participants in such market of unbearable loss. Even if United States has a long history of option trading with a relatively mature mechanism, the pricing of option is still a major topic in financial world with a lot of questions. In spite of some pricing criteria like put-call parity and Black-Sholes model, the pricing of options in real world is still difficult to be fully understood and calculate precisely. That is because the pricing of options not only depends on the database we can easily access, but also on the relationship between supply and demand, and the volatility of the underlying assets in the future, which are variables that we confused with. Since we cannot change others' decisions and determine the future volatility, it is sophisticated to calculate the certain price of the options. However, since the uncertainty, investors could purchase certain options based on their own prediction on future trend. As the result, the arbitrage opportunities and trading strategies based on such uncertainty deserve to be studied to generate large profit in the capital market.

In this essay, we will focus on the 2,097,150 options given with stock index as the underlying assets in Chicago Board Options Exchange (CBOE) in January 2017. The reason why we use data given in CBOE relies on many aspects. Basically, the data is easily accessed and already well organized, which saves a lot of time for us. Secondly, considering CBOE has been doing option trading for almost 50 years, so the trading mechanism is as mature as possible compared with other similar exchanges. Hence, we choose data in CBOE as our database to see whether it has any pricing violations and seek for some possible improvement while other immature mechanisms can simply imitate the existing pricing mechanism of CBOE.

To test whether the prices of options in CBOE are proper, we will consider the following aspects which would lead to arbitrage: boundary violation, convexity and exercise price violation, the relationship between option lives and prices violation, put-call parity and Black-Sholes model (BSM) together with implied volatility. With some certain arbitrage strategies, we would calculate the frequency and the expectation of the magnitude of the profits and loss through the data provided to check the violations. Moreover, the variation of the profits in accordance to moneyness of the options, time to maturity and stock price would also be taken into account to make sure an accurate pricing system.

After a complete analysis stated before, we come to some simple conclusions. First for boundary analysis, we found the violation rate of the options is, to some degree, sensitive to the stock price. The frequency of breaking the upper bound has a negative correlation with the corresponding stock price while the frequency of breaking the lower bound has a positive correlation. For convexity and time to maturity, since most of the violations are controlled within a small amount of profit, it is less possible to generate money. However, the violations of exercise price are considered relatively serious with the reason that the reported prices have an average of 86.89% of deviation from options' theoretical prices, and more than one-third of the options deviating more than 100% compared with the theoretical price. With reference to put-call parity and BSM violation, they are both frequently detected and should be seriously considered. And for BSM, most of the pricing errors of call options appear with larger moneyness, while most of the pricing errors of put options appear with smaller moneyness.

Based on all calculation, call options appear to be easier to generate profit with high frequency and large profit interval. Also, some recommendations have be put to pay more attention on exercise price conditions, put-call parity and BSM violations.

In the work, we offer an analytical expression on the detection of mispricing. Analysis and trading strategies are also clearly declared in the following paper. Given the data and findings we get, some suggestions are also made to pursue higher profit arbitrage opportunities.

2. Literature Review

The option returns, to some extent, depend on the risks that the investors take to purchase this option. Coval & Shumway (2000) broke the traditional option pricing methods based on the prices of the underlying assets, instead, opened the eye to focus on the nature of the option returns which is highly correlated to the risks. Through their research, they examined the long-run option returns in the context of implications set forth by asset pricing theory like CAPM and BSM and found that expected call/put returns exceed/below the risk-free rate and increase with the strike price. What's more, both call and put contracts earn exceedingly low returns based on their level of systematic risk. Besides, expected option returns move linearly with option betas. Since they believe that option risks should be priced in standard asset pricing theory, the results of the research showed that there is something like systematic stochastic volatility other than systematic risk is also priced in option contracts.

However, there is no research done in their paper to show strong evidence systematic stochastic volatility can be perfectly fit in pricing model and how it fits in.

In addition to risks taken by investors, momentum of stock market also takes a role in option pricing model. Amin, Coval, & Seyhun (2004) test the predictions of the standard option pricing models according to whether there would be some relation between the option prices and the stock market momentum. Since the perfect capital market assumption cannot be achieved in reality, the option prices may not be fully replicated by other portfolios. As the results, some realistic option prices would deviate from theoretical prices. Such deviation was explained in text by checking if S&P 100 index option prices (OEX) depend on past market returns. Based on the data collected, they found that the past returns on option models had an important influence on option prices, which leads to efforts to resolve the observed deviation in option prices through no-arbitrage-based option pricing models less possible to succeed.

Therefore, they insisted that past stock returns should be included in the functions of systematic variation in implied volatilities of options. It is the first time that people systematically examine the relation between momentum and option pricing rather than simple literature. It emphasized the influence of past stock market momentum on option prices and offered a more precise method for investor to invest in capital market.

Based on the above papers, the importance of implied volatility in option pricing models has been clearly declared. Figlewski and Green (1999) also showed that the volatility parameters is very important which may bring large model risk based on its uncertainty. However, according to the findings from Amin, Coval, & Seyhun (2004), the implied volatility is difficult to be determined due to a lot of uncertain factors. As the results, how the option return moves in accordance to volatility deserved a further study. Gao and Han (2013) presents a robust finding that the average delta-hedged option return is negative and decreases monotonically with an increase in the idiosyncratic volatility of the underlying stock. Instead of a simple reflection of known patterns on cross-section stock return which is under perfect markets, their finding is consistent with market imperfections and constrained financial intermediaries. Cao et al. (2018) also tested the relation between future delta-hedged equity option returns and volatility of volatility (VOV) according to the methods of implied volatility, EGARCH volatility from daily returns and realized volatility from high-frequency data, to study how volatility influence the delta-hedged equity option returns. Through all the data generated, negative correlation was found between future delta-hedged equity option returns and VOV. Also, their findings suggest that a higher premium should be charged for single-name options with higher uncertainty of volatility as it is more difficult to be hedged.

Apart from implied volatility, some efforts of seeking for alternative estimate of implied volatility should also be put forward to improving the option pricing model since many limitations may cause the implied volatility deviated from the market's true volatility forecast (Fleming, 1998). With strategies of straddles portfolios and delta-hedging based on the difference between historical realized volatility and at-the-money implied volatility, Goyal & Saretto (2011) thought that one should view the portfolios sorts as sorts on option prices with decile one (ten) representing over-(under-)priced options which is independent of the validity of the Black and Scholes (1973) model. They conjecture that with this method, the investor would get higher profit compared with simply using implied volatility.

Regardless of the limitations of implied volatility, the methods of detecting mispricing and how to exploit the mispricing are also deserved more efforts. Since the empirical researches on options are highly dependent on published stock and option quotations, many mispricing can be detected due to the non-simultaneity of the option and stock quotations (Bookstaber, 1981). Goyal & Saretto (2011) suggest that deviations between historical realized volatility and implied volatility estimates are important sources of determining volatility mispricing. By this finding, a zero-cost trading strategy, long (short) in the portfolio with a large positive (negative) difference in these two volatilities, can produces a significant average monthly return.

3. Data and methodology

3.1 Assumption

Based on the standard theories of option's pricing framework, such as Black-Sholes model (1973) and put-call parity, perfect capital market is considered as the basic assumption which means no extreme conditions, no transaction costs and unlimited short should be allowed. At the same time, (instantaneous) common stock returns are assumed to be normally distributed with reference to Black-Sholes model (1973).

To make the models simple to understand, in this paper we simply consider all the models to be continuous. This is a little different from reality since the real market is based on bid-ask criteria which pricing model should be discrete in fact. However, bid-ask rule suggests that the transactions are recorded only if the transactions have taken place. While our paper only test on theoretical level, meaning that the transactions can happen anytime during option life. Considering all the reasons stated, we think continuous model is more appropriate in our paper.

Since the model is built on the economic level, we assume that frequency of violations has no influence on the later violation to simplify our model, which means the correlation between the frequency of violations and the probability (number) of later violations is zero.

Considering the trading rule in CBOE, $0.5 transaction cost would be taken in consideration.

3.2 Data Collection

In this paper, 2,097,150 options given with stock index as the underlying assets was obtained from CBOE. Through all the data, 1,048,575 are calls while the remaining 1,048,575 are puts. Based on the same number of calls and puts, we divided them into different segments with different underlying assets. In this way, we can easily generate the convexity with simple tools, like EXCEL.

According to the past studies, the upper bound and lower bound restrictions have been shown must exist in option market regardless of the kind of the underlying assets and market situation (Galai, 1978). As the result, boundary analysis was proved to be an efficient way to avoid simple arbitrages in real market and the reason would be shown by some simple counter examples afterwards.

Taking dividend situations into consideration, each call and put must satisfy the following conditions (Rodriguez, 2003):

Lower bound: the price of the option should be higher than its intrinsic value. If not, arbitrage opportunities may arise. For example, if the price of the option is lower than its intrinsic value, which means that the cost of buying an option is less than the benefit of the current option. At this time, we can buy the option and exercise it at the maturity based on the rule of European Options so that we simply lock the profit, which is the difference between the price of the option and its intrinsic value.

Upper bound: for a call option, its price must not exceed the corresponding underlying assets; while for a put option, its price must be lower than the corresponding present value of theexercise price. Otherwise, arbitrage opportunities may occur. Also taking a call option for example, if its price is higher than the corresponding stock price, then the buyer could buy the stock directly instead of buying the option to lock the profit. The same situation also holds true for put options.

3.3 Convexity violations

To test more precisely, some mathematical properties of option price are applied in model in convexity analysis. The convexity of option price can be written as a function of exercise price (Kijima, 2002). Suppose that the exercise price is the independent variable x, and the call premium is the corresponding variable y, then they must hold a function relationship $y = f(x)$.

To avoid the existence of simple arbitrage opportunities, the efficient option should be convex which means the second derivative of the price function should be positive. Otherwise, the butterfly spread can be used to gain risk free profit (Ekström and Tysk, 2009).

In this case of arbitrage, delta-hedge strategy is necessary to be applied in order to eliminate the sensitivity of change of stock price. Typically, while we trying to exploit the arbitrage opportunity by long (short) a call option, delta shares of stock should be sold (bought) in order to achieve a delta-neutral situation (Howell, 2008). With the whole process though the delta-hedge strategy, total cashflow would be $6c \times S - \mathbf{t}$ $(\mathbf{t} - 6c \times S)$ Same strategy can also be applied to put options.

Among 1,048,575 groups of call options, 3,033 groups did not meet the upper bound while 713 groups did not meet the lower bound, accounting for 0.291% and 0.068% respectively. Among all violations, there was no call option violating the upper and lower bounds at the same time. So the total number of mispricing of call options is 3,746. Generally speaking, the number of violations of the upper bound is much larger than that of the lower bound.

For 1,048,575 sets of put options, 2,758 groups did not meet the upper bound while 445 groups did not meet the lower bound, accounting for 0.263% and 0.043% respectively. Besides, no data was shown to violate the upper and lower bounds at the same time, accounting for 0%. The total number of violations of put options is 3,203 and the number of violations of the upper bound is also larger than that of the lower bound.

It can be seen from the table that there were many pricing errors in the aspect of flexibility, but the magnitude of violations is not large. For call options, there are 31.83% pricing errors, of which 57.65% are controlled within 5%; for put options, there are 25.18% pricing errors, of which 55.72% are controlled within 5%. Since the capital market cannot be perfect in reality, this can be easily explained. As we should pay a certain amount of transaction fee when we try to exploit the arbitrage opportunities, this means the mispricing with ratio below 5% may not be bring cash inflows in real world.

Table 1 Degree of mispricing based on the theoretical price

Ratio	0%-5%	5%-20%	20%-50%	50%-100%	>100%
Call Options	57.65%	16.01%	8.08%	5.20%	9.35%
Put Options	55.72%	26.61%	14.32%	3.73%	0.01%

We also noted that options with pricing irregularities of more than 100% are all high-risky activities in which the difference between the exercise price and the stock price is very large (generally double or even triple).

One interesting finding is that the violation rate of convexity was basically evenly distributed on the date, but it dropped slightly on Fridays. In other words, options are more accurate when it came closely to the end of the week. But this phenomenon is not significant.

Considering difficulty on comparing the origin database, we strictly select 801,718 call options and 580,911 put options in order to ensure the simplicity of the algorithm and the preciseness of the conclusion. All of samples chosen are strictly controlled.

The frequency and percentile of mispricing are shown as follows:

Table 2 Time to maturity violations of call and put options

Time to Maturity	Correct	Incorrect	Invalid	Total
Call Conditions Frequency	593096	208621	246857	1048575
Percentile	56.56%	19.90%	23.54%	100.00%
Put Conditions Frequency	457179	123732	46764	1048575
Percentile	43.60%	11.80%	44.60%	100.00%

The preliminary results showed that in the selected call options, the pricing error accounts for 25.88%; for the put options, the pricing error accounts for 21.3%.

Table 3 Some features of options

Items of Call Options	Correct	Incorrect
Mean Value of Stock Price	100.410667	60.47209906
Standard Deviation of Stock Price	159.5082865	84.16412114
Coefficient of Variation of Stock Price	**1.588559181**	**1.391784351**
Mean Value of Strike Price	96.60791447	56.37157146
Standard Deviation of Strike Price	156.6140884	89.98059462
Coefficient of Variation of Strike Price	**1.621131035**	**1.596205184**
Mean Value of Call Price	13.24371648	11.86272945
Standard Deviation of Call Price	34.67109238	20.48785107
Coefficient of Variation of Call Price	**2.617927712**	**1.727077327**
Mean Value of Maturity	77.35008498	37.51073957
Standard Deviation of Maturity	93.76564643	49.50234097
Coefficient of Variation of Maturity	**1.21222422**	**1.319684483**

With further observation on the mean value and standard deviation of each option, we found that the group with wrong pricing has the characteristics of smaller mean value and smaller standard deviation compared with those correct pricing. Based on such phenomenon, we did some further analysis.

In this part, the violation range is relatively large, especially for the part over 100%. For the mispricing of call options, the maximum is 759%, 34.59% is more than 100%.At the same time, for the mispricing of put options, even if the range of violations is still large, it is relatively small compared with call options. In this case, almost 40% of violations are controlled within 5%, but about 30% are still more than 100%, and the maximum is 25099%. Among all these violation from call and put options, 57.32% were detected from the situation that the option price is less than $1, and 32.67% comes from the situation that the difference between the exercise price and the corresponding stock price is too large (double or more).

4. Conclusion

This paper mainly tests the boundaries, convexity, time to maturity, strike price, put-call parity conditions and BSM violations of the options, and demonstrates the mispricing situation in CBOE in January 2017. The pricing errors of both call and put options were analyzed in the terms of the standards of stock price, moneyness and the magnitude of violations.

For the boundary conditions, regardless of the types of the options (call option or put option), the violation frequency of breaking the upper bound is far higher than breaking the lower bound. Among them, the main pricing error comes from the options with relatively low-priced stock as underlying assets. With the increase of the corresponding stock price, the frequency of breaking the upper bound decreased significantly, while the frequency of breaking the lower bound grew gradually. In moneyness, the mispricing of call options mainly comes from the high moneyness, while put options, on the contrary, mainly comes from the low moneyness. Most of the violations are within 5%, so it can be considered that the violation of boundaries conditions is not serious.

For convexity conditions, one third of the call options and one quarter of the put options violated rules. In terms of moneyness, most of the pricing errors of call options come from higher moneyness, while the pricing errors of put options are very evenly distributed with respect to moneyness. Actually, no matter how moneyness changes, the frequency of violations of put options hardly changes. More than half of the pricing errors are controlled within 5% and the vast majority are controlled within 20%. In general, it can be considered that the behavior of violating the convexity condition is not serious.

For time to maturity, less than a fifth of options have violated this rule. The frequency of violations decreased with the increase of stock price. At the same time, the frequency of violations increases rapidly with the decrease of maturity, which means that the closer to the exercise date, the greater the probability of pricing errors. More than half of the violations are limited to 5%, and there are no options with more than 100% violations. Therefore, it can be considered that the violation of maturity conditions is not serious.

In conclusion, based on our finding, more violations are found on call options except for strike price violation. Also, we recommend more attention should be put on exercise price conditions, put-call parity and BSM violations.

References

[1] Amin, K., Coval, J. and Seyhun, H., 2004. Index Option Prices and Stock Market Momentum. *The Journal of Business*, 77(4), pp.835-874.

[2] Black, F. and Scholes, M., 1973. The Pricing of Options and Corporate Liabilities. *Journal of Political Economy*, 81(3), pp.637-654.

[3] Bookstaber, R., 1981. Observed Option Mispricing and the Nonsimultaneity of Stock and Option Quotations. *The Journal of Business*, 54(1), p.141.

[4] Cao, J., Vasquez, A., Xiao, X. and Zhan, X., 2018. Volatility Uncertainty and the Cross-Section of Option Returns. *SSRN Electronic Journal*,.

[5] Chung, K., Park, S. and Ryu, D., 2015. Trade Duration, Informed Trading, and Option Moneyness. *SSRN Electronic Journal*,.

[6] Coval, J. and Shumway, T., 2000. *Expected Option Returns*. University of Michigan Business School.

[7] Ekström, E. and Tysk, J., 2009. Bubbles, convexity and the Black–Scholes equation. *The Annals of Applied Probability*, 19(4), pp.1369-1384.

[8] Figlewski, Stephen, and T. Clifton Green, 1999, Market risk and model risk for a financial institution writing options, *Journal of Finance* 54, 1465-1499.

[9] Fleming, J., 1998. The quality of market volatility forecasts implied by S&P 100 index option prices. *Journal of Empirical Finance*, 5(4), pp.317-345.